Announcements

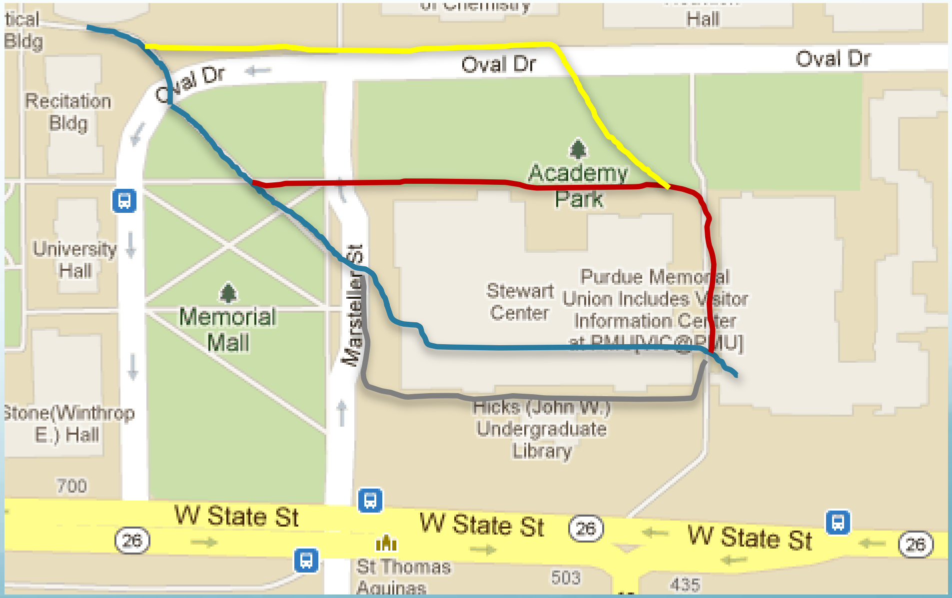
- Midterm next week Monday
- No class next Thursday
- Review this Thursday

New Course Spring 2013!

- CS 290 00 Contemporary Issues
- Robb Cutler
  - 3 credits
  - No prerequisites
  - No programming

More than one way to solve a problem

- There is always more than one way to solve a problem.
  - You can walk different paths from A to B.
- Some solutions are better than others.
- Need metric so we can we compare them…



Our programs (functions) implement algorithms

- *Algorithms* are descriptions of computations for solving a problem:

  To find the min element in a list

  1. Initialize min to the first element
  2. Scan the list
  3. For each element, if it is smaller than min, update min

- Programs (functions for us) are executable interpretations of algorithms:

```
min = L[0]
for x in L:
    if x<min: min = x
```

- The same algorithm can be implemented in many different languages and on many different platforms.

How do we compare algorithms?

- For example, there is more than one way to search.
  - How do we compare algorithms to say that one is faster than another – independent of the hardware platform?

- Computer scientists use something called *Big-O notation*
  - It's the *order of magnitude* of the running time of an algorithm with large input size

- Big-O notation ignores the differences between languages, even between compiled vs. interpreted, as well as hardware speed.
  - It focuses on how the *number of steps* to be executed grows
  - It focuses on *scalability*

What question are we trying to answer?

- If I am given a larger problem to solve (larger input), how is the performance of the algorithm affected?

    - If I change the input, how does the running time change?

How can we determine the complexity?

- Step 1: we must determine the "input," or what data the algorithm operates over, and its size, measured fairly
  - Testing if n is prime has input size proportional to $\log(n)$

- Step 2: determine how many operations are needed to be done for each piece of the input, measured fairly
  - Determining the smallest element in an unordered list of size n is not constant-time $O(1)$ – it is $O(n)$

- Step 3: eliminate constants and smaller terms for big O notation

E.g., searching a phone book…

- In the first week we introduced the task of searching a phone book as an example algorithm. Compare 3 algorithms:
- Algorithm 1:
  - Start at the front, check each name one by one, until found or coming to the end of the book
- Algorithm 2:
  - Use an index to jump to the correct letter, then search sequentially, as before, starting at that point
- Algorithm 3:
  - Split in half, choose which half has the name, repeat recursively until name found -- or size is zero and the name is not found

Algorithm 1

```
# a phone_book is a list of entries
# an phone book entry is [name,number]
# we are searching for key_name

def algorithm1(phone_book, key_name):
    for k in range(len(phone_book)):
        if key_name == phone_book[k][0]:
            return phone_book[k][1]
    return
```

- Worst case: key_name not in book or close to the end
  - Computational work ~ len(phone_book)
  - We say algorithm1 is O(n) where n is len(phone_book)
  - Each time through loop is O(1) -- constant time

Algorithm 2

```
# a phone_book is a list of entries
# an phone book entry is [name,number]
# we are searching for key_name

inx1 = first location of name starting key_name[0]
inx2 = last location of  name with that letter
def algorithm2(phone_book, key_name, inx1, inx2):
    for k in range(inx1,inx2+1):
        if key_name == phone_book[k][0]:
            return phone_book[k][1]
    return
```

- Worst case: key_name not in book and lots of names with that first letter
  - Computational work ~ # of names with that letter which is ~ n
  - We say algorithm2 is also $O(n)$ where n is len(phone_book) in the worst case
  - Each time through loop is $O(1)$ -- constant time
  - There is a constant-factor speed-up immaterial to the scaling behavior

Algorithm 3

```
def algorithm3(book, key, lo, hi):
    k = (hi+lo)/2
    if book[k][0] == key: return book[k][1]
    if book[k][0] < key:
        return algorithm3(book,key,k+1,hi)
    else: return algorithm3(book,key,lo,k-1)
    if hi < lo: return   “not in book”
return
```

- Worst case: key is not in book
    - Computational work ~ $\log_2(n)$, where n is len(book)
    - We say algorithm3 is $O(\log(n))$
    - Each call is O(1) -- constant time -- except for the recursive calls

How to count steps

- Loops:
  - Body takes k steps, then loop takes km steps, where m is the number of times through the loop
- Straight-line code usually O(1) constant time
  - Exceptions:
    - Assignments, arithmetic, comparisons are O(1) unless we deal with large structures, such as lists or very long strings
    - Function calls estimated separately

Nested loops are multiplicative

```
def loops():
 count = 0
 for x in range(1,5):
  for y in range(1,3):
   count = count + 1
   print (x,y,"--Ran it",count,"times”  )
```

```
>>> loops()
1 1 --Ran it  1 times
1 2 --Ran it  2 times
2 1 --Ran it  3 times
2 2 --Ran it  4 times
3 1 --Ran it  5 times
3 2 --Ran it  6 times
4 1 --Ran it  7 times
4 2 --Ran it  8 times
```

Big-O notation ignores constants

- Consider if we executed a particular statement 3 times in the body of the loop
  - If we execute each loop 1 million times this constant becomes meaningless (ie: n = 1,000,000)
  - For large n, $O(n^2) \equiv O(3n^2)$

```
def loops(n):
  count = 0
  for x in range(1,n):
    for y in range(1,n):
      count = count + 1
      count = count + 1
      count = count +1
```

Lets compare our phone book search algorithms

- Algorithm 1:
  - Start at the front, check each name one by one
    - O(n)
- Algorithm 2:
  - Use the index to jump to the correct letter
    - O(n/26) … O(1/26 * n) … O(n)
- Algorithm 3:
  - Split in half, choose which half must have the name, repeat until found
    - O(log n)

More on big O notation

- http://en.wikipedia.org/wiki/Big_Oh_notation
  - Additional background
- We mentioned that big O notation ignores constants
  - Lets look at this more formally:
  - Big O notation characterizes functions (algorithms in our case) by their growth rate

Identify the term that has the largest growth rate

Num of steps	growth term	asympt. complexity
$6n + 3$	$6n$	$O(n)$
$2n^2 + 6n + 3$	$2n^2$	$O(n^2)$
$2n^3 + 6n + 3$	$2n^3$	$O(n^3)$
$2n^{10} + 2^n + 3$	2^n	$O(2^n)$
$n! + 2n^{12} + 2^n + 3$	$n!$	$O(n!)$

CQ: For large n, which is faster?

A. $10^{20}n$

B. $10^{-20}\,n^2$

CQ: For large n, which is faster?

A. $10^{20} n$ (seconds)

B. $10^{-20} n^2$ (seconds)

A is better when $n > 10^{20}$

Comparison of complexities: fastest to slowest

- $O(1)$ – constant time
- $O(\log n)$ – logarithmic time
- $O(n)$ – linear time
- $O(n \log n)$ – log linear time
- $O(n^2)$ – quadratic time
- $O(n^3)$ – cubic time
- $O(2^n)$ – exponential time
- $O(n!)$ – factorial time

Do we know of any O(1) algorithms?

- These are “constant time” algorithms
- Simple functions that contain no loops are usually O(1)
- Examples:
  - project 1 computation;
  - “real feel” of temperature at certain humidity

Finding something in the phone book

- O(n) algorithm
  - Start from the beginning.
  - Check each page, until you find what you want.
- Not very efficient
  - Best case: One step
  - Worst case: n steps where n = number of pages
  - Average case: n/2 steps

What about algorithm 2?

- Recall that algorithm 2 for finding a name in the phone book used the index
- Can we make this algorithm faster by having an index for each letter?
- Would such an algorithm have a lower running time than our binary search?
  - Would it have a lower complexity?

Clicker Question

- What is the complexity of hiding the image in project 3, where the image is $n \times n$ pixels?

A. $O(1)$

B. $O(n)$

C. $O(n^2)$

D. $O(n^3)$

Not all algorithms are the same complexity

- There is a group of algorithms called *sorting algorithms* that place things (numbers, names) in a sequence.
- Some of the sorting algorithms have complexity around $O(n^2)$
  - If the list has 100 elements, it takes about 10,000 steps to sort them.
- However, others have complexity $O(n \log n)$
  - The same list of 100 elements would take only 460 steps.
- Think about the difference if you're sorting your 1,000,000 customers…

We want to choose ~~the~~ algorithm with the a best complexity

- We want an algorithm which will be
  - **Fast: a "lower" complexity mean an algorithm will perform better on *large* input**
  - Stable: gives accurate answers
  - Space efficient
  - Easy to implement and maintain

Generating the dragon curve

```
F(m,ch):
    if m==1: return ch
    return F(m-1,'R') + ch + F(m-1,'L')
```

- Key question to ask: are the strings of negligible length?
- So we need to solve a recurrence:

$$T(1) = 1$$
$$T(m+1) = 2T(m) + 1$$

which is solved by

$$T(m) = 2^m - 1$$

- Proof by induction⋯

Homework

- Study for the midterm

Announcements

- Midterm on Monday, Nov. 5, 8pm, WTHR 200

Midterm 2 Review

- Many advanced issues are explained in the text, part 2

- Important control structures
  - Functions
  - Loops
  - Conditionals
  - Recursion

- Important things to review
  - Binary numbers, bit operations
  - Boolean operators (and, or, not)
  - String operations: len, ord, +, *, slice, index, strip, split, etc.
  - List operations: +, *, slice, index, assign, append, insert, etc.
  - Libraries standard, os, url
  - Input/output
  - Tree and matrix encodings, operations

Functions

- Functions allow us to "name" a region of code in a similar fashion that a variable allows us to name a value
- Functions provide us a mechanism to reuse code

```
def name(input) :
    code to execute when function is called
    return (output)
```

Local vs Global Variables

- Functions introduce a new “scope”
  - This scope defines the lifetime of local variables
  - The scope is the function body

```
def name(input) :
    code to execute    <--- scope of local variables
    return output
```

Important Concepts

- Only ONE return is ever executed
  - The return ends execution of the function
  - If there are statements after the executed return they are ignored!
- If there is no return that is executed, or if a return is executed without a value, then the function returns the special python value: None
- The return specifies the value that the function "outputs"
  - If you return a variable the function outputs the value stored in that variable

Conditionals

- Conditionals allow us to test for a condition and execute based on whether the condition is True or False

```
if condition :
    code to execute if condition is True
else:
    code to execute if condition is False
```

Things to remember

- The else clause is optional
- There MUST be a condition to check after an elif
    - The else clause is still optional here too
- Anything we can express with elif we can express with a nested if

```
if x < 10:
    print( “Hello” )
elif y > 30:
    print( “World” )
```

```
if x < 10:
    print( “Hello” )
else:
    if y > 30:
        print( “World” )
```

Python Convention

- Python interprets non-Boolean expressions when they appear in a conditional:

```
if x:
    <statements>
```

- [ ], 0, “” are all considered False
- Nonempty lists, nonempty strings, nonzero numbers are understood as True

Lists, Strings

- Use bracket notation to access elements [ ]
- Lists and Strings use an index to access an element
  - We consider such structures ordered (as opposed to sets which would be unordered)

Creating Structures

- Lists – use the [ ] notation
  - List = [1, 2, 3, 4, 5, “foo”]
- Strings use the single or double quotes, or triple repeated quotes
  - String = “this is my string”

Indexing

- X[k] means:
  - Element k+1 in a list
  - Character k+1 in a string
- If lists are nested, we can refer to them by multiple indexing from outside in:

```
X = [1,2, [3,[4]],5]
X[2] == [3,[4]]
X[2][1] == [4]
X[2][1][0] == 4
```

Negative indexing

- For S a string or list:
  - S[0] == S[-len(S)]
  - S[1] == S[-len(S)+1]
  - …
  - S[len(S)-1] == S[-1]

Structures can contain other structures

- Lists can contain elements which themselves are Lists
- This is used in encodings:
  - Matrix encoding
  - Tree encoding

Specialized Structures built from structures containing structures

- Matrices
  - Represented as a list of lists
  - The internal lists are either *rows* or *columns*

- Trees
  - Represented as an arbitrary nesting of lists
  - The structure of the elements represents the parent node and the *branching* of it. Leaves are simple values.

Matrices

- Review matrix multiplication
- Review how to populate a matrix
    - Go through the pre lab examples
- Review how to create a matrix
    - Python short hand

Traversing a Matrix

- Is B encoded column by column or row by row?
  - We do not know
  - …. But what if I told you this loop prints the matrix row by row?

```
B = [[1, 0, 0], [0.5, 3, 4], [-1, -3, 6], [0, 0, 0]]
for j in range(3):
  for i in range(4):
      print (B[i][j])
```

How do we find out how the matrix is encoded?

- Step one: figure out the order in which the values are printed
  - 1, 0.5, -1, 0, 0, 3, -3, 0 , 0, 4, 6, 0
- Step two: compare this to the matrix
  - B = [[1, 0, 0], [0.5, 3, 4], [-1, -3, 6], [0, 0, 0]]
- Step three: deduce the encoding by comparing the order to the matrix
  - The matrix is encoded column by column!

Applying the same intuition to matrices

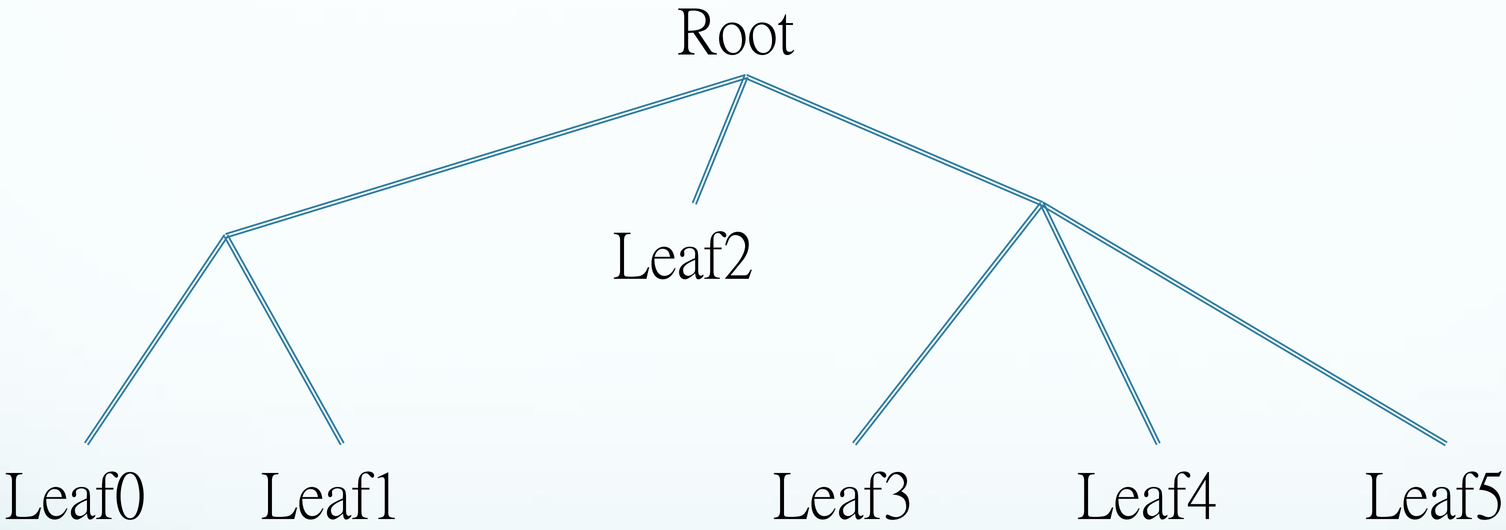
- Lets traverse the matrix the other way!

```
B = [[1, 0, 0], [0.5, 3, 4], [-1, -3, 6], [0, 0, 0]]
for j in range(3):
  for i in range(4):
      print (B[i][j])
for i in range(4):
  for j in range(3):
      print (B[i][j])
```

Trees

- Know how to select elements from a tree
- Know how to encode a tree using (nested) lists
- Distinguish internal nodes and leaves
- Understand how to visit tree nodes recursively

Given the picture can you generate the python list?

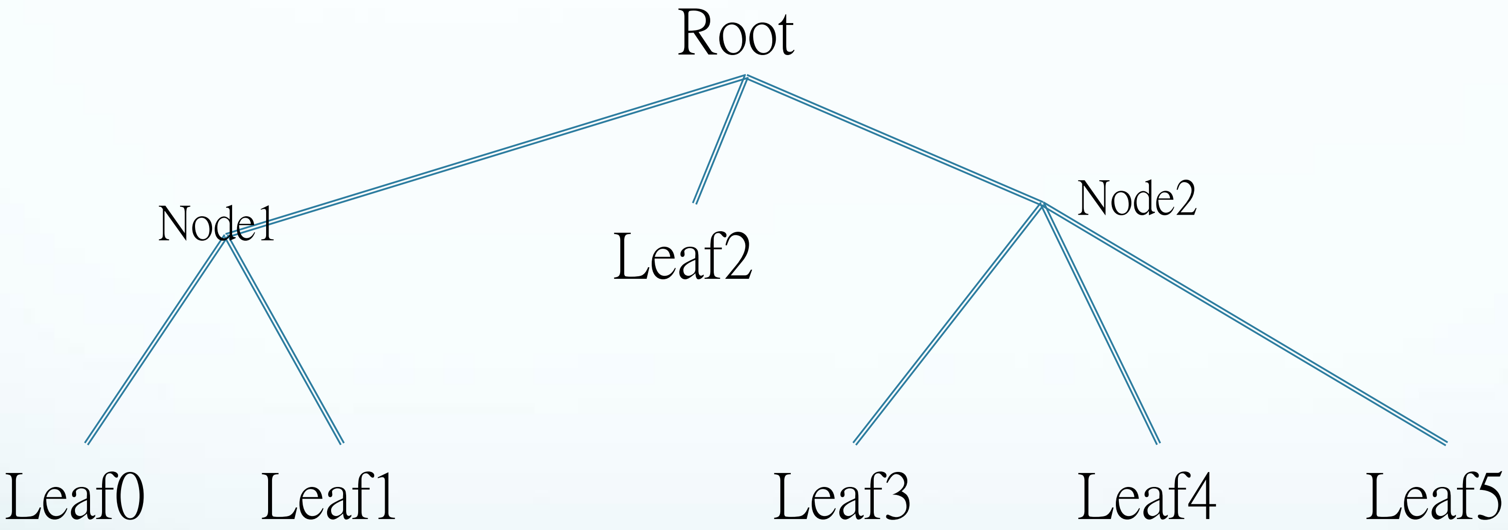


Root

Leaf2

Leaf0 Leaf1 Leaf3 Leaf4 Leaf5

Given the picture can you generate the python list?



Root

Node1

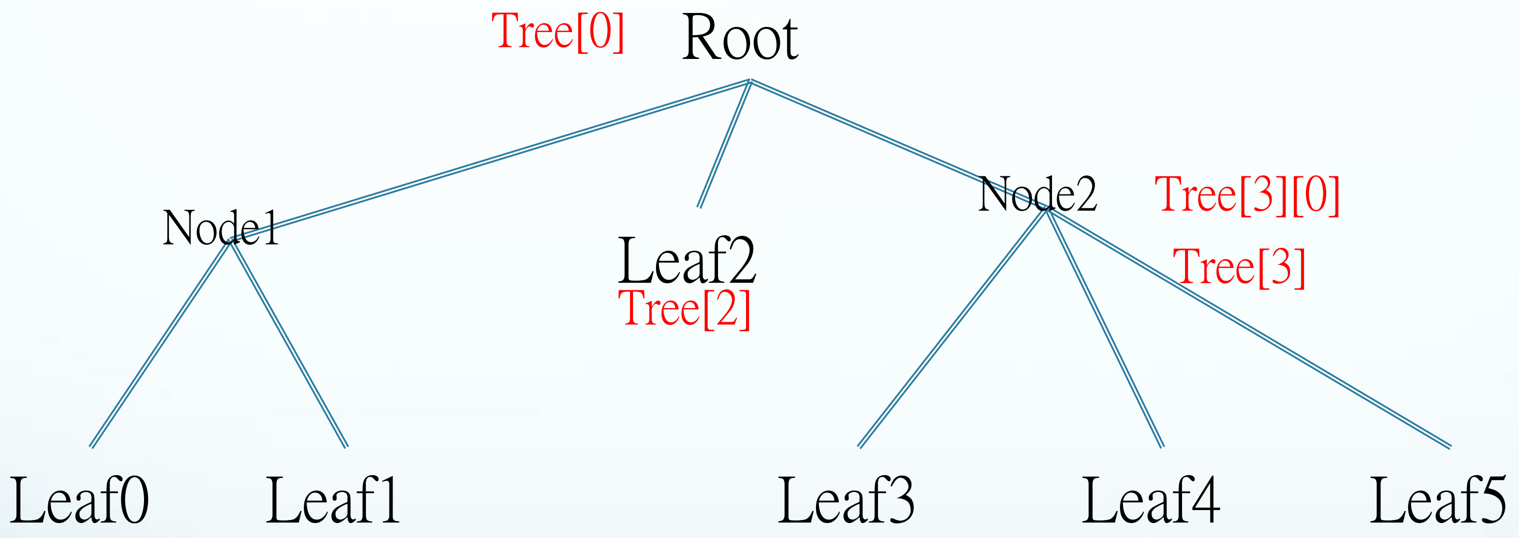
Leaf2

Node2

Leaf0 Leaf1 Leaf3 Leaf4 Leaf5

Tree = [Root, [Node1 , Leaf0, Leaf1], Leaf2, [Node2 , Leaf3, Leaf4, Leaf5]]

Indices provide us a way to “traverse” the tree



Tree[0] Root

Node1

Leaf2
Tree[2]

Node2 Tree[3][0]

Tree[3]

Leaf0 Leaf1 Leaf3 Leaf4 Leaf5

Tree = [Root, [Node1 , Leaf0, Leaf1], Leaf2, [Node2 , Leaf3, Leaf4, Leaf5]]

Library Modules

- urllib
    - Allows us to get data directly from webpages
- os
    - Allows us to manage files and interact with the operating system

File I/O

- What is the difference between the various modes?
  - We saw in class “w” “r”
- What is the difference between read, readline, and readlines?

CQ: read() ends with

1. A) ends with ‘\n’
2. B) ends with character just before ‘\n’

Methods on Files

- object.method() syntax: this time files are our object
  - Example: file = open(“myfile” , “w”)
- file.read() -- reads the entire file as one string
- file.readlines() – reads the file as a list of strings
- file.write() – allows you to write to a file
- file.close() – closes a file

Immutable Structures

- Strings are considered immutable
  - What does this mean in practice?
  - We cannot assign new values to the indexed elements in strings

- Errors for strings:
  - A = “mystring”
    A[3] = “p”

Strings and Parsing

- What are the most important operations?
  - find
  - rfind
  - split
  - strip
  - rstrip
  - slicing
  - upper
  - lower

String.find

- string.find(sub) – returns the lowest index where the substring sub is found or -1
- string.find(sub, start) – same as above, except searching the slice string[start:]
- string.find(sub, start, end) – same as above, except searching the slice string[start:end]

String.rfind

- string.rfind(sub) – returns the highest index where the substring sub is found or -1
- string.rfind(sub, start) – same as above, except using the slice string[start:]
- string.rfind(sub, start, end) – same as above, except using the slice string[start:end]

String.split

- String.split(delimiter) breaks the string String into parts, separated by the delimiter
  - print (“a b c d” .split(“ “))

Would print: [‘a’ , ‘b’ , ‘c’ , ‘d’]

Concrete Example

```
foo = ” there their they’ re”

elem = foo.split(" ” )

for i in elem:

   print(i.split( “e"))
```

```
[ ‘there’ , ‘their’ , “tey’ re” ]
```

```
[ ‘th', ‘r’ , ‘’ ]

[ ‘th', ‘ir']

[ ‘th', “y’ r” , ‘’ ]
```

String.strip

- “hello helpful handy hammer” .strip(‘ehr”) results in

  “llo helpful handy hamm”

Manipulating Strings

- How might we reverse a string?

- We used the same technique for the problems
  - Build up a new string piece by piece

Example: Reversing Strings

```
def reverse(str):
    output = "”
    for i in range(0, len(str)):
        output = output + str[len(str)-i-1]
    print (output)
```

or do the loop this way:

```
    for ch in str:
        output = ch + output
```

Useful things to know

- range function
- ord and chr functions
- int and type functions
- recursion